The relation between spatial skills and early number knowledge: The role of the linear number line

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Evidence that spatial skills support math and science achievement usually comes from assessments with explicit spatial content such as geometry and engineering (e.g., Shea, Lubinski, & Benbow, 2001). Here we propose that spatial skills may also be crucial for success in domains that are less overtly spatial. Specifically, we suggest that spatial skills play a role in the math tackled in early elementary school, as children begin to form a mental representation of numbers. Children's ability to represent numbers linearly on a number line is a critical skill that is related to other skills such as arithmetic calculation (e.g., Booth & Siegler, 2008). We propose that individual differences in early spatial skills influence how quickly children develop a linear mental representation of numbers, and that this representation in turn helps children to succeed on numerical tasks that do not require overt spatial representations.

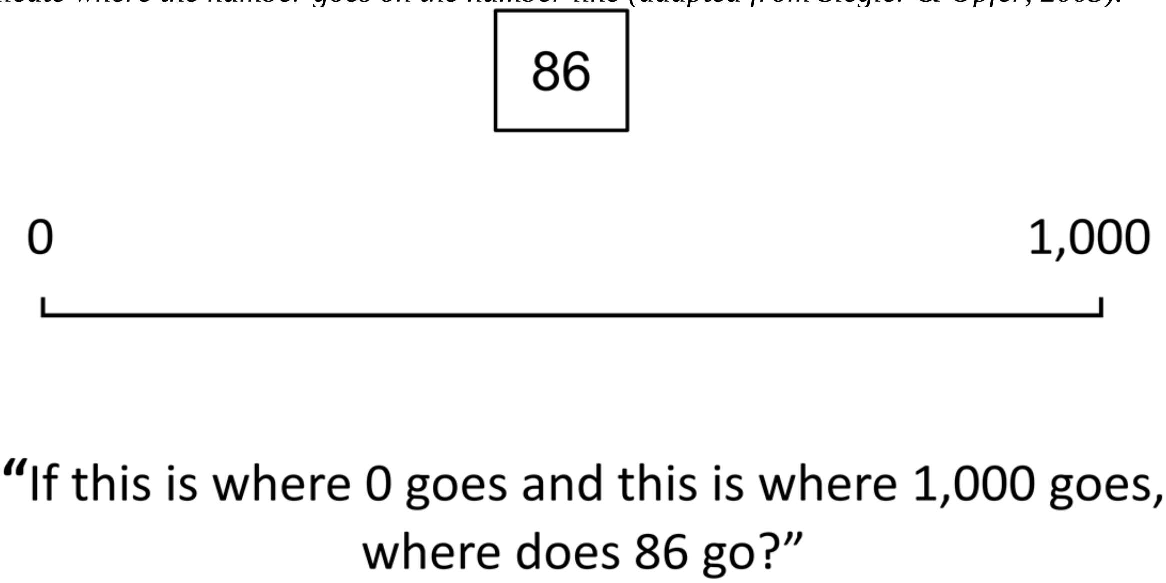
To support our claims, we present data from two longitudinal studies. First, we show that children's general spatial skills predict improvement of their linear representation of numbers over time. Second, we show that children's early spatial skills predict later non-spatial numerical skills and, moreover, that linear number line representations account for (mediate) this relation. Our results begin to explain how and why spatial skills are related to children's mathematics achievement even in math domains that are not overtly spatial in nature.

Study 1

First- and second-grade students' (N=152, 81 girls, 71 boys) spatial skills, number line knowledge, and math and reading achievement were assessed in a one-on-one session with an experimenter during the first three months of the school year. Students' number line knowledge was assessed again 6 months later.

Number line knowledge was assessed using the 0-1,000 number line task (Siegler & Opfer, 2003). Children were shown a number line (see Figure 1) and the experimenter said, "This number line goes from 0 at this end to 1,000 at this end. If this is 0 and this is 1,000,where would you put N?" The experimenter then held up a card displaying an Arabic numeral, and children responded by drawing a hatch mark through the number line to show the position of that number. Children's score was based on the linearity of their response (the percent of variance explained (R^2) by the best-fitting linear model relating children's responses to the number requested).

Figure 1. Graphic depiction of the number line task. The child must draw a hatch mark to indicate where the number goes on the number line (adapted from Siegler & Opfer, 2003).



Spatial skills were assessed using the mental rotation subtest of Thurstone's Primary Test of Mental Abilities (Thurstone, 1938). *Math and reading achievement* were assessed as control measures, using the Applied Problems and Letter-Word Identification subtests of the nationally-normed Woodcock-Johnson III Tests of Achievement (Woodcock, McGrew, & Mather, 2001).

We found that children with higher levels of spatial skill at the beginning of the school year had larger gains in number line knowledge over the course of the year. That is, controlling for children's initial number line knowledge, children's initial spatial skills were a significant predictor of end-of-year number line knowledge ($\beta=.20$, $t=2.57$, $p<.05$). In addition, controlling for children's initial scores on standardized tests of math and reading achievement did not change this relation. This suggests that general spatial skills that are involved in representing, maintaining and transforming visual representations may facilitate the formation of the linear mental number line.

Study 2

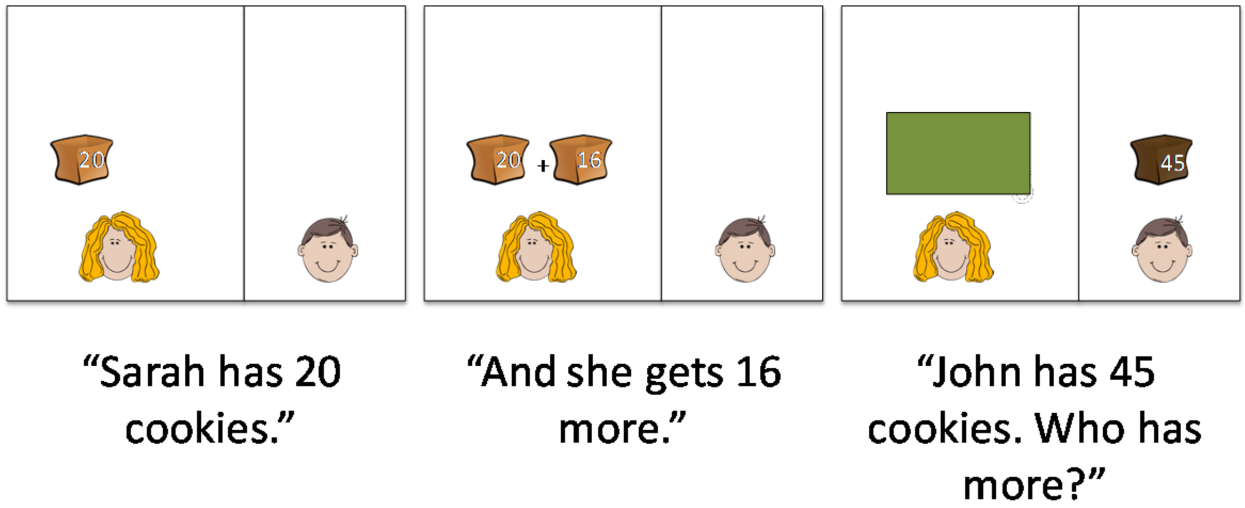
Forty-two children (25 male, 17 female) participated in Study 2 as part of a larger longitudinal study of child development.

Spatial skills were assessed at age 5.4 (SD=0.2) using the Children's Mental Transformation Task (CMTT; Levine, Huttenlocher, Taylor, & Langrock, 1999). This task requires children to choose which shape would be made by moving two separate pieces together. The CMTT is appropriate for younger children than the Thurstone mental rotation task used in Study 1, but is otherwise similar in content and presentation format.

Number line knowledge was assessed at age 6.3 (SD=0.4) using the 0 to 100 number line task (Siegler & Opfer, 2003), as described in Study 1.

Approximate symbolic calculation skill was assessed at age 8.0 (SD=0.4) using a computerized task in which number symbols are presented and then hidden, and children indicate which side of the screen has a larger total (see Figure 2; adapted from Gilmore, McCarthy, & Spelke, 2007).

Figure 2. Graphic depiction of the approximate symbolic calculation task. Images are presented sequentially and children must respond by saying whether Sarah or John has more cookies.

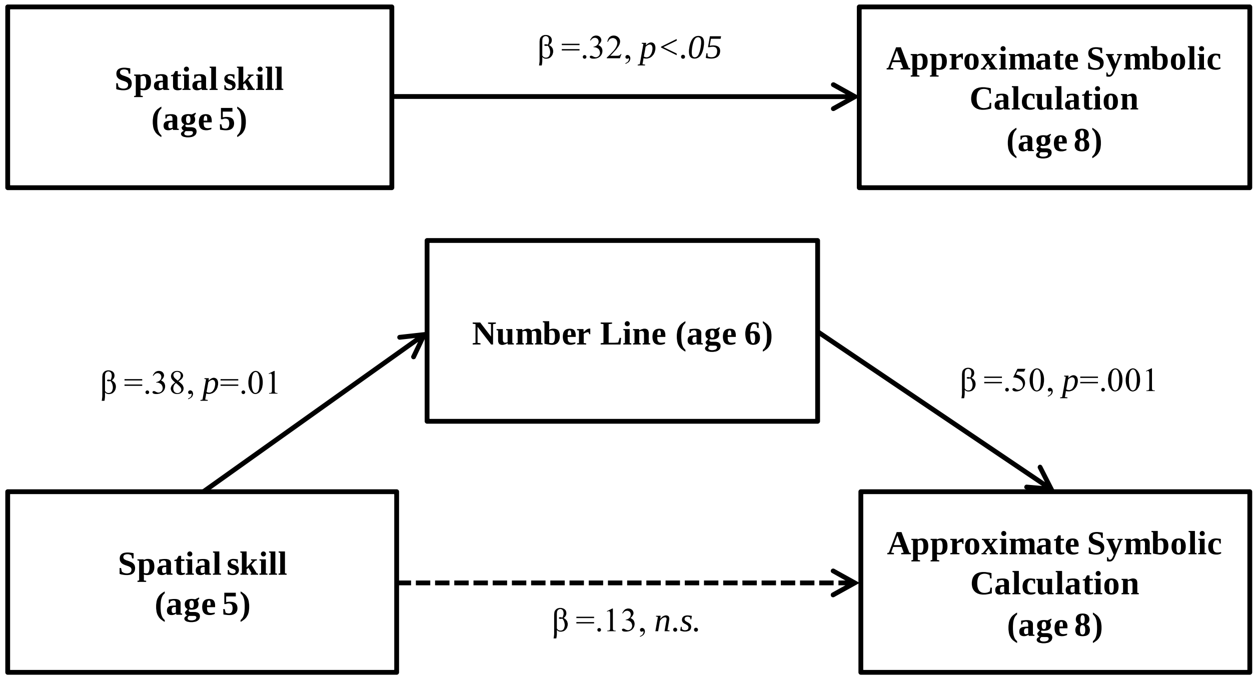


Vocabulary knowledge was measured as a control variable at age 6.2 (SD=0.6) using the Peabody Picture Vocabulary Test (PPVT-III; Dunn & Dunn, 1997). The PPVT-III requires children to choose one of 4 pictures that matches an orally-presented word.

Our main question was whether children's early spatial skills (age 5) predicted their later approximate symbolic calculation skill (age 8), and if so, whether this relation was mediated by children's number line knowledge at an intermediate time point (age 6). To test this mediation hypothesis, we conducted a series of regressions, controlling for children's vocabulary knowledge at age 6 in all models.

As shown in Figure 3, children's spatial skills at age 5 predicted their approximate symbolic calculation score at age 8 ($\beta=.32$, $t=2.21$, $p<.05$). In addition, children's spatial skill at age 5 predicted their number line knowledge at age 6 ($\beta=.38$, $t=2.72$, $p=.01$). Finally, when number line knowledge and spatial skills were entered simultaneously as predictors of approximate symbolic calculation score, number line knowledge remained a significant predictor ($\beta=.50$, $t=3.44$, $p=.001$) while spatial skills did not ($\beta=.13$, $t=0.91$, $p>.10$). Thus, number line knowledge mediates (accounts for) the relation between spatial skills and approximate symbolic calculation.

Figure 3. Mediation analysis based on series of regression models (N=42). All models control for vocabulary knowledge at age 6.



Discussion

Results from two longitudinal studies converge to reveal a strong relationship between spatial skills, number line knowledge, and math achievement. Both studies are consistent with a causal model in which spatial skills facilitate children's development of a linear numerical representation, which in turn helps them to succeed on non-spatial numerical tasks. Thus, improving children's spatial skills may have positive impacts on their future success in STEM disciplines not only by improving the spatial skills that are necessary in many science and engineering fields, but also by enhancing the numerical skills which form the backbone of the advanced mathematics critical to all STEM fields.

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